Overview of Mixed Models

Eduard Ponarin

Higher School of Economics

Laboratory for Comparative Social Research

January 22, 2015

A two-level model

Compared to the regular linear model, this model's intercept is *random*, that is it varies by *group*, the second-level unit of analysis.

- $y_{ij} = \alpha_j + \beta x_{ij} + \epsilon_{ij}$

A two-level model

Compared to the regular linear model, this model's intercept is *random*, that is it varies by *group*, the second-level unit of analysis.

- $y_{ij} = \alpha_j + \beta x_{ij} + \epsilon_{ij}$
- $\alpha_j = \gamma_0 + \eta_j$

In this particular model, second-level variation is left unexplained.

A two-level model

Compared to the regular linear model, this model's intercept is *random*, that is it varies by *group*, the second-level unit of analysis.

- $y_{ij} = \alpha_j + \beta x_{ij} + \epsilon_{ij}$
- $\alpha_j = \gamma_0 + \eta_j$

In this particular model, second-level variation is left unexplained.

- $y_{ij} = \gamma_0 + \eta_j + \beta x_{ij} + \epsilon_{ij}$

Empty Model (simple ANOVA Model)

- $y_{ij} = \alpha_j + \epsilon_{ij}$

Empty Model (simple ANOVA Model)

- $y_{ij} = \alpha_j + \epsilon_{ij}$
- $\alpha_j = \gamma_0 + \eta_j$

Empty Model (simple ANOVA Model)

- $y_{ij} = \alpha_j + \epsilon_{ij}$
- $\alpha_j = \gamma_0 + \eta_j$
- $y_{ij} = \gamma_0 + \eta_j + \epsilon_{ij}$

Intraclass Correlation Coefficient

Definition

$$ICC = \frac{\sigma_{\alpha}^{2}}{\sigma_{\alpha}^{2} + \sigma_{\epsilon}^{2}} \quad (1)$$

This is the share of group variance in the overall variance of Y. Thus it describes how different the groups are. It also describes how strongly units in the same group resemble each other.

Fixed Slopes and Random Intercepts

- $y_{ij} = \alpha_j + \beta x_{ij} + \epsilon_{ij}$
- $\alpha_j = \gamma_0 + \eta_j$
- $y_{ij} = \gamma_0 + \eta_j + \beta x_{ij} + \epsilon_{ij}$

Compared to the empty model, this model would reduce ϵ-variance but would not reduce α-variance much (in this case, α-variance is identical to η-variance).

FSRI with a second-level predictor

- $y_{ij} = \alpha_j + \beta x_{ij} + \epsilon_{ij}$
- $\alpha_j = \gamma_0 + \gamma z_j + \eta_j$
- $y_{ij} = \gamma_0 + \gamma z_j + \eta_j + \beta x_{ij} + \epsilon_{ij}$

This model would explain some of both ϵ-variance and α-variance (no longer the same as η-variance). The mixed equation can be re-written as follows in order to separate the fixed part and the random part (the last two terms):

$y_{ij} = \gamma_0 + \gamma z_j + \beta x_{ij} + \eta_j + \epsilon_{ij}$

Random intercepts and random slopes

- $y_{ij} = \alpha_j + \beta_j x_{ij} + \epsilon_{ij}$
- $\alpha_j = \gamma_{00} + \eta_{0j}$
- $\beta_j = \gamma_{10} + \eta_{1j}$
- $y_{ij} = \gamma_{00} + \eta_{0j} + (\gamma_{10} + \eta_{1j})x_{ij} + \epsilon_{ij}$

The mixed equation can also be written as follows, separating the fixed part and the random part (the last three terms):

$y_{ij} = \gamma_{00} + \gamma_{10} x_{ij} + \eta_{0j} + \eta_{1j} x_{ij} + \epsilon_{ij}$

The term $\eta_{1j} x_{ij}$ models heteroscedasticity since error obviously depends on the level of x.

Cross-level interaction effect

- $y_{ij} = \alpha_j + \beta_j x_{ij} + \epsilon_{ij}$
- $\alpha_j = \gamma_{00} + \gamma_{01} z_j + \eta_{0j}$
- $\beta_j = \gamma_{10} + \gamma_{11} z_j + \eta_{1j}$
- $y_{ij} = \gamma_{00} + \gamma_{01} z_j + \eta_{0j} + (\gamma_{10} + \gamma_{11} z_j + \eta_{1j}) x_{ij} + \epsilon_{ij}$

The mixed equation can also be written as follows, separating the fixed part and the random part (the last three terms):

$$y_{ij} = \gamma_{00} + \gamma_{01} z_j + \gamma_{10} x_{ij} + \gamma_{11} z_j x_{ij} + \eta_{0j} + \eta_{1j} x_{ij} + \epsilon_{ij}$$

This report was presented at the training methodological workshop
"Economic and Social Changes: values effects across Eurasia”.

March 31 - April 6, 2015 – Turkey.

http://lcsr.hse.ru/en/seminar_m2015

Настоящий доклад был представлен на методологическом учебном семинаре
«Экономические и социальные изменения: оценка эффектов по всей Евразии».

31 марта – 6 апреля 2015 года – Турция.

http://lcsr.hse.ru/seminar_m2015